

# A Semi-Probabilistic Safety-Checking Format for Seismic Assessment of Existing RC Buildings

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**ABSTRACT:** The present work focuses on probability-based safety-checking formats suitable for implementation in codes for the purpose of the assessment of existing buildings. As an alternative to the current approach in the European code based on applying the confidence factors, a semi-probabilistic approach is proposed based on applying a bias factor and a dispersion factor inside safety checking formats for both static and dynamic analyses. In this work, an efficient simulation-based Bayesian method is used in order to estimate the factors for the safety-checking formats for the case-study structure for different knowledge levels and different outcomes of the tests and inspections. It can be argued that, if the so-called bias factor and the dispersion factor can be calibrated for various building types and different outcomes of tests and inspections, they may offer an acceptable alternative to confidence factors in lieu of rigorous case-specific probability-based assessment of existing buildings.

## 1 INTRODUCTION

The performance assessment of existing buildings is particularly important in the process of decision-making for retrofit, repair and re-occupancy of existing buildings. With respect to new construction, the seismic assessment of existing buildings is characterized by uncertainties in the structural modeling parameters. The current European code-based procedures seem to account for them, by dividing the mean material properties by a factor larger than unity, known as the confidence factor. The confidence factor, which is classified based on discrete levels of knowledge about the building, seems to create an overall margin of safety in the performance assessments without specifically addressing the modeling uncertainties. A rigorous approach to seismic assessment of existing buildings needs to take into consideration all sources of uncertainty present in the assessment problem. The probabilistic approach to structural assessment coupled with the Bayesian updating framework is particularly suitable for taking into account in a quantifiable manner all the sources of uncertainty and all the information available about an existing building, ranging from the original design documents to the test and inspection results. In a fully probabilistic approach, the structural performance can be measured by the probability of exceeding a specified limit state. There is no general closed-form solution available for the probability of failure and it is usually calculated using numerical integration methods. In this work a semi-probabilistic safety-checking for the existing buildings is proposed. It is demonstrated how the parameters of a simplified analytic safety-checking format, arranged similar to Load-Resistance Factor Design (LRFD), for different knowledge levels (KL), can be estimated considering dynamic

analyses. This format is already adopted in the American Department of Energy Guidelines DOE-1020 and in SAC-FEMA guidelines [FEMA 2000]. For seismic assessments based on static analyses, an analytical safety-checking formulation is adopted which yields the global structural response, represented by a structural performance parameter, corresponding to a certain confidence. For each KL, the outcome of tests and inspections is incorporated by employing the Bayesian updating framework. If the parameters of these safety-checking formats (for static and dynamic analyses) are estimated based on different KL's, building types and outcome of tests, these formats would be potentially suitable for code implementation.

### 1.1 *The structural performance parameter and the structural reliability*

The structural performance parameter in this work is formatted in terms of a critical demand to capacity ratio. This parameter which is denoted as  $Y$ , assumes the value of unity on the onset of the limit state  $LS$ . In the case of static analyses, the capacity spectrum method (CSM) [Fajfar, 1999] is used to obtain  $Y$ . Moreover, at the onset of the limit state, the shear capacity of the structural components is also verified by calculating the shear demand to capacity ratio for the structural components. The overall structural performance parameter is finally taken as the larger between the critical shear component demand to capacity ratio and the overall demand to capacity ratio derived from CSM. The structural reliability in the static case is expressed as the probability that  $Y$  exceeds one  $P_f = P(Y > 1)$ .

In the case of dynamic analyses, the *cut-sets* concept in system reliability theory [Ditlevsen and

Madsen, 1996] is employed to find the critical component demand to capacity ratio that takes the structure closer to the onset of the limit state  $LS$ . This critical demand to capacity ratio corresponds to the strongest component of the weakest structural failure mechanism [Jalayer et al., 2007]:

$$Y = \min_l \max_j \frac{D_{lj}}{C_{lj}} \quad \text{Eq. 1}$$

Where  $l$  is the structural mechanism index considered and  $j$  is the component index within the  $l^{\text{th}}$  mechanism. In this case the mechanisms considered involve the ultimate chord rotation in the components, the formation of global mechanisms (e.g., soft story and beam mechanisms) and the component shear capacity. The structural reliability in the dynamic case is represented by the mean annual frequency (MAF) that the performance parameter  $Y$  defined in Eq. exceeds unity (or simply, the MAF of failure). Taking the spectral acceleration at the fundamental period of the structure as the intensity measure, the MAF of failure can be calculated by integrating fragility and hazard for all values of spectral acceleration [Jalayer et al., 2007]:

$$\lambda_f = \lambda_{Y>1} = \int P(Y > 1 | S_a) | d\lambda(S_a) | \quad \text{Eq. 2}$$

### 1.2 A performance-based safety-checking format (dynamic analyses)

As an alternative to the CF approach in code-based recommendations, a probabilistic and performance-based approach, adopted in the American Department of Energy Guidelines DOE-1020 and in SAC-FEMA guidelines [Cornell et al., 2002], is chosen in this work. This simplified approach leads to an analytical and closed-form solution which compares the factored demand against factored capacity. The factored demand and capacity are respectively equal to median demand and capacity multiplied by some factors. The magnifying demand factors and the de-magnifying capacity factors can take into account all sources of uncertainty, such as record-to-record (ground motion) variability, structural modeling uncertainty and the uncertainty in the capacities. This approach, that is also known as the *Demand and Capacity Factor Design* (DCFD) [Cornell et al., 2002] for its similarity with LRFD, takes into account the overall effect of the various types of uncertainties on a global structural performance parameter. An alternative representation of the DCFD format, which is employed in this work, compares the factored structural performance parameter  $Y$  against unity:

$$\eta_Y(P_o) \cdot e^{\frac{1}{2b}(\beta^2_{Y|S_a} + \beta^2_{UC})} \leq 1 \quad \text{Eq. 3}$$

Where  $P_o$  is an acceptable threshold for structural failure probability and  $\eta_Y(P_o)$  is the median structural performance parameter corresponding to the acceptable probability  $P_o$ .  $k$  is the slope coefficient for linear regression (in the logarithmic space) of spectral acceleration hazard versus spectral acceleration and  $b$  is the slope coefficient for linear regression (in the logarithmic space) of the structural performance parameter  $Y$  versus spectral acceleration. The terms  $\beta_{Y|S_a}$  and  $\beta_{UC}$  represent the effect of ground motion (GM) variability and structural modeling uncertainties, respectively, on the total dispersion in the structural performance parameter given spectral acceleration; (see Jalayer and Cornell [2009] for more details on how to estimate  $\eta_Y(P_o)$  and  $\beta_{Y|S_a}$ ). The inequality in Eq. can be verified with a certain  $x\%$  confidence:

$$\eta_Y(P_o) \cdot e^{\frac{1}{2b}\beta^2_{Y|S_a}} \leq e^{-\Phi^{-1}(x)\sqrt{\beta^2_{Y|S_a} + \beta^2_{UC}}} \quad \text{Eq. 4}$$

Where  $\Phi^{-1}(x)$  is the inverse Gaussian cumulative distribution function (CDF) for percentile  $x$ . Note that in this formulation, in contrast to Eq. , the factored demand is compared to a less than unity quantity in order to provide a certain level of confidence in the assessment, which is suitable for code implementation.

### 1.3 A performance-based safety-checking format (static analyses)

In the static case, safety-checking is performed by calculating a given percentile  $x\%$  of the structural performance parameter  $Y$  and by verifying whether it is less than or equal to unity:

$$\eta_Y \cdot e^{\Phi^{-1}(x)\beta_Y} \leq 1 \quad \text{Eq. 5}$$

Where  $\eta_Y$  is the median value and  $\beta_Y$  is the standard deviation of the logarithm for the structural performance variable  $Y$ . As explained before, the percentile  $x$  reflects a desired level of confidence in the structural performance.

## 2 METHODOLOGY

### 2.1 Characterization of the uncertainties

It is assumed that the vector  $\underline{\theta}$  represents all the uncertain parameters considered in the problem. This work focuses on the uncertainty in the structural modeling parameters related to the available infor-

mation on the characteristics of existing buildings. This is the type of uncertainty that is believed to be addressed implicitly by the application of CF's. Two groups of structural modeling uncertainties are considered, the uncertainty in the mechanical property of materials and the uncertainty in the structural construction details. In particular, the structural construction details can include, stirrup spacing, concrete cover, anchorage and splice length; these are also known as the *defects*. In order to take into account the uncertainty in the representation of the GM, a set of 30 records based on Mediterranean events are chosen from European Strong Motion Database, 28 recordings, and the database of the Next Generation Attenuation of Ground Motions (NGA) Project, 2 recordings. They are all main-shock recordings and include only one of the horizontal components of the same registration. The soil category on which the GMs are recorded is stiff soil ( $400 \text{ m/s} < V_{s30} < 700 \text{ m/s}$ ) which is consistent with the Eurocode 8 soil-type B (the soil-type for the site of the case-study presented in this work). The earthquake events have moment magnitude between 5.3 and 7.2, and closest distances ranging between 7 km and 87 km.

The parameters identifying the prior probability distributions for the material mechanical properties (concrete strength and the steel yielding force) have been based on the values typical of the post world-war II construction in Italy [Verderame et al. 2001a,b]. Table 1 shows these parameters that are used to define the Lognormal probability distributions for the material properties.

Table 1. The uncertainties in the material properties (systematic per floor).

Material	Type	Median	COV
$f_c$	L	165 kg/cm <sup>2</sup>	0.15
	N		
$f_y$	L	3200 kg/cm <sup>2</sup>	0.08
	N		

The prior probability distributions for the structural detailing parameters are defined based on qualitative prior information coming from expert judgment or based on *ignorance* in the extreme case [Jalayer et al., 2010]. Table 2 shows (for illustrative purpose only) the example specifications used to construct the prior probability distributions for the structural detailing parameters. It shows a list of possible defects, their probability distribution and correlation characteristics (arbitrarily set, although they may be defined statistically based on practitioner surveys).

## 2.2 Updating the probability distributions

The probability distributions for the structural modeling parameters are updated employing the Bayesian framework for inference. It is assumed that the material properties are homogeneous across each floor or construction zone. Therefore, the material property value assigned to each floor can be thought of as an average of the material property values across the floor in question. The results of tests and inspections for each floor are used to update the probability distribution for the mean material property across the floor.

Table 3. The uncertainties in structural detailing parameters.

Defects	Possibilities	Prob.	Type
Insufficient anchorage (Beams)	sufficient (100% effective)	Uniform [0.50,1]	Systematic over floor
	Absent (50% effective)		
Error in diameter (Columns)	$\phi_{16}$	Uniform [0.77, 1]	Systematic over floor and section type
	$\phi_{14}$		
Superposition (Columns)	100% of the area effective 75% of the area effective	Uniform [0.75,1]	Systematic over floor
Errors in configuration (columns)	More plausible configuration	Uniform [0.75,1] [0.67,1]	Systematic over floor and section type
	Less plausible configuration		
Absence of a bar (beams)	Absence of a bar	Uniform [0.70,1] [0.69,1] [0.60,1]	Systematic over floor and section type
	Presence of a bar		
Stirrup spacing	Uniform (beams)	Uniform [15, 30]cm	Systematic
Stirrup spacing	Uniform (beams)	Uniform [20, 35]cm	Systematic
spacing of shear rebar	Uniform (column)	Uniform [20, 35]cm	Systematic

## 2.3 An efficient method for estimation of reliability

In this work, an efficient simulation-based Bayesian method is used in order to estimate the structural reliability based on a small number simulations (around 10-30). This efficient method is described herein.

Suppose that the probability of failure is described by an analytical probability distribution with parameters  $\chi = (\eta_Y, \beta_Y)$  (e.g., median and standard deviation of the Lognormal distribution). If the probability of failure given the set of parameters is denoted by  $P(F|\chi)$ , the expected value for the probability of failure for a given set of data values  $\mathbf{d} = \{d_i : i = 1 : N\}$  is expressed as:

$$E[P(F | \mathbf{d})] = \int_{\Omega} P(F | \chi) p(\chi | \mathbf{d}) d\chi \quad \text{Eq. 6}$$

where  $p(\boldsymbol{\chi} | \mathbf{d})$  is the posterior probability distribution for the set of parameters  $\boldsymbol{\chi}$  given the data  $\mathbf{d}$  and  $\Omega$  is the space of possible values for  $\boldsymbol{\chi}$ . Likewise, the variance for the probability of failure is calculated as:

$$\sigma^2_{P(F|\mathbf{d})} = E[P(F | \mathbf{d})^2] - E[P(F | \mathbf{d})]^2 \quad \text{Eq. 7}$$

In particular, if the dataset  $\mathbf{d}$  is expressed in terms of a set of  $Y$  values calculated for different realizations of the uncertain parameters within the problem, the structural reliability or the probability of failure in the case considering structural modeling uncertainties (given the code-specified spectrum) can be expressed by a Lognormal complementary cumulative distribution function (CCDF) as following  $\boldsymbol{\chi} = \{\eta_Y, \beta_Y\}$ :

$$P(Y(\underline{\theta}) > 1) = 1 - \Phi\left(\frac{-\log \eta_Y}{\beta_Y}\right) \quad \text{Eq. 8}$$

Where,  $\Phi$  is the Gaussian CDF,  $Y$  is the structural performance index and  $\eta_Y$  and  $\beta_Y$  are the median and the standard deviation (of the logarithm) for the probability distribution of the structural performance index. Using the Bayesian updating framework, the posterior probability distribution for median and standard deviation based on data  $Y$  is written as [Box and Tiao, 1992]:

$$P(\eta_Y, \beta_Y | Y) = k \beta_Y^{-(n+1)} \exp\left(-\frac{v s^2 + n(\log \eta_Y - \overline{\log Y})^2}{2\beta_Y^2}\right) \quad \text{Eq. 9}$$

$$k = \sqrt{\frac{n}{2\pi}} \left(\frac{\Gamma(v/2)}{2}\right)^{-1} \left(\frac{v s^2}{2}\right)^{v/2}$$

Where  $Y = \{Y_1, \dots, Y_n\}$  is the vector of  $n$  different realizations of the structural performance index,  $k$  is a normalizing constant,  $\Gamma(\cdot)$  is the gamma function,  $v = n - 1$ ,  $\overline{\log Y}$  is the sample mean value for  $\log Y$  and  $v s^2$  is sum of the squares of the deviations from the sample mean value. The expected value and the standard deviation for the probability of failure are calculated from Eq. 7 and Eq. 8 based on the posterior probability distribution  $p(\eta_Y, \beta_Y | Y)$  in Eq. 9.

In the dynamic case, the structural fragility as a function of spectral acceleration in the presence of modeling uncertainties and uncertainties in the representation of the GM can be calculated from the following Lognormal CCDF:

$$P(Y(\underline{\theta}) > y | S_a) = 1 - \Phi\left(\frac{\log y - \log \eta_{Y|S_a}}{\beta_{UT}}\right) \quad \text{Eq. 10}$$

$$\beta^2_{UT} = \beta^2_{Y|S_a} + \beta^2_{UC}$$

Where  $\eta_{Y|S_a}$  is the median for the probability distribution of the structural performance index and  $\beta_{UT}$  is the total standard deviation for the probability distribution of the structural performance index including the contribution from record-to-record variability and the overall effect of the structural modeling uncertainties. The terms  $\beta_{Y|S_a}$  and  $\beta_{UC}$  represent the effect of the uncertainty in the GM representation and the uncertainty in the material properties and the structural details, respectively. It should be noted that Eq.10 yields the structural fragility; after integrating it with the hazard function for the spectral acceleration, the MAF that the structural performance variable  $Y$  exceeds a specific value, or the structural risk curve, is obtained.

Suppose that a selection of  $n$  ground motion records are used to represent the effect of GM uncertainty on the structural performance index. Let  $S_{a,i}$  and  $Y_i$  represent the spectral acceleration and the performance index for the GM record  $i$ , respectively. The posterior probability distribution for standard deviation is calculated as:

$$P(\beta_{UT} | Y) = \left[\frac{1}{2} \Gamma\left(\frac{v}{2}\right)\right]^{-1} \left(\frac{v s^2}{2}\right)^{\frac{v}{2}} \beta_{UT}^{-(v+1)} \exp\left(\frac{-v s^2}{2\beta_{UT}^2}\right) \quad \text{Eq.11}$$

The data pairs  $(Y, S_a)$  are gathered by calculating  $Y$  for the set of  $n$  GM records applied to the structural model generated by different realizations of material mechanical properties and structural detailing parameters.  $v$  is the degrees of freedom and is equal to  $n - 2$ ,  $v s^2$  is equal to the sum of the square of the residuals for a linear regression of  $\log Y$  on  $\log S_a$  and  $a$  and  $b$  are the regression coefficients. The joint posterior probability distribution for the coefficients of the linear regression  $\boldsymbol{\omega} = (\log a, b)$  are calculated as:

$$P(\boldsymbol{\omega} | Y, S_a) = k \left[1 + \frac{(\boldsymbol{\omega} - \hat{\boldsymbol{\omega}})^T \mathbf{X}^T \mathbf{X} (\boldsymbol{\omega} - \hat{\boldsymbol{\omega}})}{v s^2}\right]^{-\frac{n}{2}} \quad \text{Eq. 12}$$

$$k = \frac{\Gamma\left(\frac{n}{2}\right) \sqrt{n \sum \log S_{a,i}^2 - (\sum \log S_{a,i})^2}}{v s^2 \Gamma\left(\frac{1}{2}\right)^2 \Gamma\left(\frac{n}{2} - 1\right)}$$

which is a *bivariate t-distribution* where  $\mathbf{X}$  is a  $n \times 2$  matrix whose first column is a vector of ones and its second column is the vector of  $\log S_{a,i}$ ,  $\boldsymbol{\omega}$  is the  $2 \times 1$  vector of regression coefficients  $\log a$  and  $b$  and  $\hat{\boldsymbol{\omega}}$  is the vector  $(\log a, b)$  of the coefficients of the linear regression of structural performance parameter  $Y$  versus  $S_a$  in the logarithmic space (due to GM variability only). The median and the standard deviation for the probability distribution for  $Y|S_a$  are taken equal to the maximum likelihood estimates  $\eta_{Y|S_a} =$

$aS_a^b$  and  $\beta_{Y|S_a=s}$ , that is, the conditional median value for  $Y$  is estimated by a power-law function of  $S_a$  and the conditional standard deviation (of the logarithm) of  $Y$  given  $S_a$  is assumed to be constant. The robust estimates for the expected value and the standard deviation of the failure probability are obtained from Eq. 6 and Eq. 7 based on the product of the posterior probability distributions  $p(\boldsymbol{\omega}|Y, S_a)$  and  $p(\beta_{UT}|Y, S_a)$  in Eq. 11 and Eq. 12, assuming they are independent  $\boldsymbol{\chi}=(\boldsymbol{\omega}, \beta_{UT})=(\log a, b, \beta_{UT})$ .

#### 2.4 Estimating the parameters of the analytic safety-checking formats

This sub-section discusses how the SAC-FEMA safety-checking format and the confidence interval formulation described in sub-sections 1.3 and 1.4 are modified and how their corresponding parameters can be estimated using the efficient Bayesian method. In the static case, the formulation in Eq. for obtaining the  $x$  percentile of the structural performance parameter is re-written as following:

$$\hat{Y} \cdot \gamma \cdot e^{\Phi^{-1}(x)\beta_Y} \leq 1 \quad \text{Eq. 13}$$

Where  $\gamma$  is a bias factor and  $\beta_Y$  is the standard deviation of the structural fragility curve.  $\hat{Y}$  represents the structural performance parameter calculated for the structural model corresponding to the median material properties based on the test results and nominal values for the structural detailing parameters. The bias factor  $\gamma$  represents the (usually larger-than-unity) factor that once multiplied by the nominal value  $\hat{Y}$  leads to the median value  $\eta_Y$ . Comparing with Eq. ,  $\gamma$  can be calculated as:

$$\gamma = \frac{\eta_Y}{\hat{Y}} \quad \text{Eq. 14}$$

Likewise, when the uncertainty in the GM representation is considered, the formulation in Eq. can be re-written as:

$$\hat{Y} \cdot \gamma \cdot e^{\frac{1}{2} \frac{k}{b} \beta_{Y|S_a}^2} \leq e^{-\Phi^{-1}(x) \sqrt{\beta_{Y|S_a}^2 + \beta_{UC}^2}} \quad \text{Eq. 15}$$

Where  $\gamma$  is a bias factor and  $\beta_{UC}$  represent the over-all effect of structural modeling uncertainties.  $\hat{Y}$  represents the structural performance parameter calculated based on the median material properties obtained from the test results and nominal values for the structural detailing parameters. For instance,  $\hat{Y}$  can be calculated by performing linear least squares as a function of the first-mode spectral acceleration based on the set of records. The bias factor  $\gamma$  represents the factor that once multiplied by the nominal value  $\hat{Y}$  leads to the median value  $\eta_Y(P_o)$

for the structural performance parameter for an admissible probability value  $P_o$ :

$$\gamma = \frac{\eta_Y(P_o)}{\hat{Y}} \quad \text{Eq. 16}$$

Where  $\eta_Y(P_o) = a S_a(P_o)^b$  using the maximum likelihood estimates of regression coefficients ( $\log a, b$ ) obtained from the probability distribution in Eq. 12 (i.e., the  $\boldsymbol{\omega}=(\log a, b)$  vector that maximizes  $P(\boldsymbol{\omega}|Y, S_a)$ ),  $S_a(P_o)$  is calculated as the spectral acceleration value corresponding to a MAF of exceedance equal to  $P_o$  on the spectral acceleration hazard curve.

### 3 NUMERICAL EXAMPLE

#### 3.1 Structural Model

As the case-study, an existing school structure located in Avellino (Italy) is considered herein. The structure is situated in seismic zone II according to the former Italian seismic guidelines [OPCM 3431, 2005]. The structure consists of three stories and a semi-embedded story and its foundation lies on soil type B. For the structure in question, the original design notes and graphics have been gathered. The building is constructed in the 1960's and it is designed for gravity loads only, as it is frequently encountered in the post second world war buildings. It is inferred from the original design notes that the steel re-bar is of the type Aq42 and the concrete has a minimum resistance equal to 165 kg/cm<sup>2</sup> (16.18 N/mm<sup>2</sup>) [R.D.L., 1939]. The finite element model of a central frame within the building is constructed assuming that the non-linear behavior is concentrated in plastic hinges located at the element ends. Each beam or column element is modeled by coupling in series of an elastic element and two rigid-plastic hinges. The rigid-plastic element is defined by its moment-rotation relation which is derived by analyzing the reinforced concrete section at the hinge location. In this study, the section analysis is based on (the widely adopted in current practice) Mander-Priestly [Mander et al., 1988] constitutive relationship for reinforced concrete, assuming that the concrete is not confined, and the reinforcing steel behavior is elastic-perfectly-plastic. The behavior of the plastic hinge is characterized by four phases, namely: rigid, cracked, post-yielding, and post-peak. In addition to flexural deformation, the yielding rotation takes into account also the shear deformation and the deformation related to bar-slip based on the code recommendations [OPCM 3431, 2005]. Moreover, the shear span used in the calculation of the plastic rotation is based on the code formulas. As it

regards the post-peak behavior, it is assumed that the section resistance drops to zero with a post-peak negative slope. The structural analyses are performed using the Open System for Earthquake Simulation (OpenSees, <http://opensees.berkeley.edu/index.php/>).

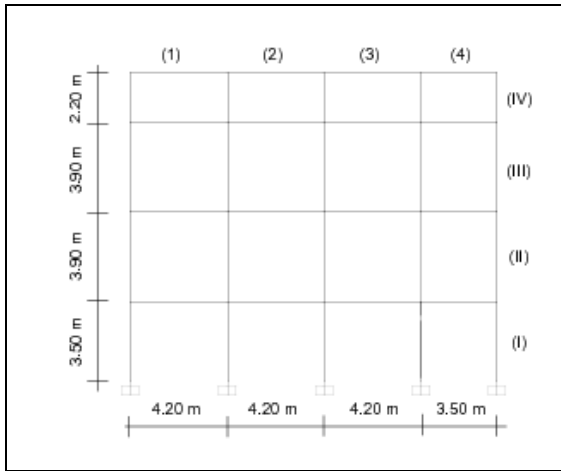


Figure 1. the central frame extracted for performing the analyses

### 3.2 Estimating the parameters of the performance-based safety-checking formats using the efficient Bayesian method

The structural fragility curve for the structure under study is calculated by employing the efficient Bayesian method described before based on  $Y$  for static analyses for a set of 20 Monte Carlo (MC) realizations of the structural model. These realizations take into account the uncertainties in the material properties and the structural defects (as listed in Tables 1 and 2). The probability distributions for the uncertain parameters are updated according to the increasing KL defined in the Eurocode 8 (CEN 2003). For each KL, 20 realizations of the structural model are generated from the (updated) probability distributions corresponding to the KL's and based on the results of in-situ tests and inspections. Since the results of tests and inspections actually available for the frame in question did not exactly match the Eurocode 8 definition of the KL's, the test and inspection results used herein are simulated based on three different simplified hypotheses: (a) 100% of the test results verify the design values indicated in the original documents (b) 50% of the test results verify the design values (c) 0% of the test results. The structural fragility for knowledge levels  $KL_1$ ,  $KL_2$  and  $KL_3$  is calculated from Eq. 6 as the expected value of the structural fragility in Eq. 8, given that its median and standard deviation are known, where the joint probability distribution for median and standard deviation is given in Eq. 9. For each KL, the standard deviation in the fragility estimate is calculated from Eq. 7 as a measure of the error in the

estimation of the structural reliability using the efficient Bayesian method.

The curves of the MAF of exceeding a given value of  $Y$  (or more concisely, the seismic risk curves) for increasing levels of knowledge are calculated by integrating the structural fragility curves, obtained from the efficient Bayesian method, and the spectral acceleration hazard curve at the site of the structure. For each KL, the fragility is calculated from Eq. 6, Eq. 10, Eq. 11 and Eq. 12 using a set of 30 MC realizations of the structural model. The conditional median  $\eta_{Y/Sa}$  and standard deviation of the logarithm  $\beta_{Y/Sa}$  for the structural performance parameter  $Y$  are estimated by employing the linear least squares of natural logarithm of  $Y$  as a function of the natural logarithm of spectral acceleration at the fundamental mode of the structure. The joint probability distribution for the linear least squares coefficients  $\omega=(\log a, b)$  is calculated from Eq. 12. The probability distribution for the standard deviation of the fragility curve  $\beta_{UT}$  (related to  $\beta_{Y/Sa}$  and  $\beta_{UC}$  through Eq. 10) is calculated from Eq. 11 based on the results of a small set of 30 MC simulations. The standard deviation as it is seen in Eq. 11 can be calculated as the square root of the sum of squares of two parts representing the effect of GM uncertainty denoted by  $\beta_{Y/Sa}$  and the structural modeling uncertainty denoted by  $\beta_{UC}$ . The set of MC realizations for each KL are generated based on the corresponding (updated) probability distributions. The suite of 30 records described in sub-section 2.1 are used. The resulting seismic risk curves are calculated (based on the three hypotheses described regarding the outcome of the tests and inspections) for knowledge levels  $KL_0$  (knowledge level before the tests and inspection results are obtained),  $KL_1$ ,  $KL_2$  and  $KL_3$ .

The fragility (and risk) curves calculated for the static and dynamic case using the efficient Bayesian method can be used in order to estimate the parameters of the analytical safety-checking formats discussed beforehand. For the static case, the bias factor  $\gamma$  is calculated from Eq. 14 and the standard deviation  $\beta_Y$  is calculated from the fragility curve obtained employing the efficient Bayesian method as half of the logarithm of the ratio of the 84<sup>th</sup> and 16<sup>th</sup> percentiles. Tables 3 and 5 outline the parameters  $\beta_Y$  and  $\gamma$  values for the three KL's considered for the case-study structure and based on static analyses. The three columns represent the three simplified hypotheses adopted previously regarding the outcome of the test results. Table 6 outlines these parameters for the knowledge level  $KL_0$  before the tests are performed. It is observed that the  $\beta_Y$  values remains

quasi-invariant with respect to the hypotheses regarding the outcome of the tests and inspections. However, they reduce as the knowledge level increases. For instance, for  $KL_0$ ,  $\beta_Y$  is close to 15% which means that based on the prior distributions considered herein, considering the structural modeling uncertainties influences the structural reliability up to 15%. The values for  $\beta_Y$  reduce to 5% for  $KL_3$ . The bias factor  $\gamma$  remains more-or-less invariant with respect to the KL; however, it changes as a function of the percentage of the test and inspection results that verify the nominal value. For example,  $\gamma$  is approximately equal to 1.40, 1.20 and 1.0 for percentages verified equal to 100%, 50% and 0%, respectively.

Table 3. Table of values for  $\beta_{UC}$  (uncertainty in the material properties and in the structural details).

		100% verified	50% verified	0% veri- fied
SPO	$KL_1$	0.0641	0.0835	0.0586
	$KL_2$	0.0527	0.0616	0.0556
	$KL_3$	0.0531	0.0554	0.0527
DYN	$KL_1$	0.0800	0.0868	0.1142
	$KL_2$	0.0393	0.0635	0.0742
	$KL_3$	0.0216	0.0472	0.0682

The observation that  $\beta_Y$  depends on the KL unlike the bias factor  $\gamma$  that remains more-or-less invariant, is somehow to be expected. That is, given a fixed percentage of the inspections results that verify the nominal values, the increase in knowledge level (i.e., the increase in the total number of inspections) is expected to reduce the dispersion in the structural performance parameter  $Y$ . On the other hand, the bias factor is expected to depend on the number of inspections that verify rather than the total number of inspections.

Table 4. Table of values for  $\beta_{UT}$  (includes the uncertainty in the material properties, the uncertainty in the structural details and the uncertainty in the ground motion representation).

		100% verified	50% verified	0% verified
DYN	$KL_1$	0.1784	0.1816	0.1938
	$KL_2$	0.1643	0.1717	0.1759
	$KL_3$	0.1609	0.1663	0.1735

For the dynamic case, the parameters for the safety-checking format in Eq. 15 are also calculated by employing the efficient Bayesian method. The bias factor  $\gamma$  is calculated from Eq. 16. The total standard deviation  $\beta_{UT} = \sqrt{\beta_{UC}^2 + \beta_{Y|S_a}^2}$  is calculated from the fragility curves obtained from the ef-

ficient Bayesian method as half of the logarithm of the ratio of the percentiles 84<sup>th</sup> and 16<sup>th</sup>, respectively. The standard deviation  $\beta_{Y|S_a}$  is estimated as the square root of the mean of the squared residuals of the regression of  $\log Y$  versus  $\log S_a$  without considering the structural modeling uncertainties (for the structural model constructed based on the median value of the test results for material properties and the nominal values for construction details). Hence, the value for  $\beta_{UC}$  is calculated as  $\beta_{UC} = \sqrt{\beta_{UT}^2 - \beta_{Y|S_a}^2}$ . Table 3 and 5 tabulate the  $\gamma$  and  $\beta_{UC}$  for different KL's and test outcomes based on non-linear time-history analyses. The same coefficients for the knowledge level  $KL_0$  are listed separately in Table 6. It is observed that the value for  $\beta_{UC}$  reduces with increasing the KL; that is,  $\beta_{UC}$  is close to 18% for  $KL_0$  and it reduces to 2% for  $KL_3$  (when 100% of the results verify) and 7% (when 0% of the results verify). The bias factor  $\gamma$  which is observed to be more-or-less invariant with respect to the KL, is approximately equal to 1.50, 1.30 and 1.0 for 0%, 50% and 100% of the test and inspections verifying the nominal tests and inspections. Table 4 outlines the estimates for  $\beta_{UT}$  for the dynamic case for knowledge levels  $KL_1$ ,  $KL_2$  and  $KL_3$  for different percentages of the test and inspection results verifying. It is observed that  $\beta_{UT}$  decreases a small amount (19%-16%) with the increasing KL. The small variation in  $\beta_{UT}$  is attributed to the fact that it includes also the dispersion  $\beta_{Y|S_a}$  due to record-to-record variability. Since the value of  $\beta_{UC}$  (around 2%-11%, depends on the KL and the percentage of the inspections verified) is small with respect to  $\beta_{Y|S_a}$  (around 16%, by definition depends neither on the KL nor on the percentage of the test results verified), the resulting  $\beta_{UT}$  values in Table 4 show little sensitivity to the KL.

It is emphasized that the values tabulated herein depend, in addition to being case-study specific, on the simplifying assumption regarding the outcome of the test results and the assumptions regarding the prior probability distributions. However, they represent an example where given the structure, the type of analysis and the outcome of the tests and inspections, the parameters of the safety-checking formats are calibrated. With regard to possible code implementations, similar tables can be obtained by characterizing the representative building types for a given location and their period of construction. The tabulated parameters can be potentially used within the safety-checking formats discussed in this work, in lieu of thorough case-specific assessments, for performance-based assessment of existing buildings.

Table 5. Table of values for the bias factor  $\gamma$ .

		100% verified	50% verified	0% veri- fied	CF
SPO	KL <sub>1</sub>	0.9933	1.2343	1.4255	<b>1.35</b>
	KL <sub>2</sub>	0.9782	1.2272	1.4294	<b>1.20</b>
	KL <sub>3</sub>	0.9701	1.2206	1.4349	<b>1.00</b>
DYN	KL <sub>1</sub>	1.0984	1.3306	1.4698	<b>1.35</b>
	KL <sub>2</sub>	1.0521	1.3046	1.4812	<b>1.20</b>
	KL <sub>3</sub>	1.0362	1.2632	1.4953	<b>1.00</b>

Table 6. Table of values for KL<sub>0</sub>.

		$\gamma$	$\beta_{UC}$
SPO	KL <sub>0</sub>	1.5245	0.1455
DYN	KL <sub>0</sub>	1.3342	0.1783

## 4 CONCLUSIONS

A semi-probabilistic safety-checking for the existing buildings is proposed. It is demonstrated how the parameters of a simplified analytic safety-checking format arranged similar to LRFD for different knowledge levels considering dynamic analyses can be estimated for a case-study structure.

In perspective, the probability-based analytical safety-checking formats calibrated for the case-study building herein, are potentially suitable candidates for implementation in the guidelines for existing buildings. It should be mentioned that in order to make accurate performance assessments, the best way to approach would be to carry out case-specific assessments based on the outcome of the tests and inspections. However, the probability-based analytical safety-checking formats and their tabulated parameters can offer significant improvements in the assessments with respect to the current CF approach; they can serve as a less-than-ideal, approximate solution with a rigorous basis.

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